Sample Mean v. OLS: *Estimation I*

1. Estimation	**Sample Mean**	**OLS**
DGM: Data Generation Mechanism	$\{Y_i\}$ iid $E(Y_i) = \mu$, $Var(Y_i) = \sigma^2$	$\{Y_i\} = \{\beta_0 + \beta_1 x_i + U_i\}$ indept. $E(Y_i \mid x_i) = \beta_0 + \beta_1 x_i$, $Var(Y_i \mid x_i) = \sigma^2$
Unknown parameter (to be estimated)	μ	β_1
Linear Estimator	$b_0 + \sum b_i Y_i$	$b_0 + \sum b_i Y_i$
Linear Unbiased Estimator	$\sum b_i Y_i \mid \sum b_i = 1$	$\sum b_i Y_i \mid \sum b_i = 0$ & $\sum b_i x_i = 1$
Variance to be minimized	$\sigma^2 \sum b_i^2$	$\sigma^2 \sum b_i^2$
Constraints	$\sum b_i = 1$	$\sum b_i = 0$ & $\sum b_i x_i = 1$

Sample Mean v. OLS: *Estimation II*

1. Estimation, continued	**Sample Mean**	**OLS**
BLUE	$b_i^* = \frac{1}{n}$	$b_j^* = \frac{(x_j - \bar{x})}{\sum (x_i - \bar{x})^2}$
Estimator	$M = \frac{1}{n}\sum Y_i$ (unweighted avg. of values)	$B_1 \mid x's = \sum \left\{ \frac{(x_i - \bar{x})^2}{(n-1)S_{xx}} \right\} \frac{(Y_i - \bar{Y})}{(x_i - \bar{x})}$ (weighted avg. of slopes)
Estimate	$\hat{\mu} = \frac{1}{n}\sum y_i$	$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_j - \bar{x})^2}$
Estimator Variance	$\frac{\sigma^2}{n}$	$\frac{\sigma^2}{\sum (x_j - \bar{x})^2}$
… Standard Deviation	$sd(M) = \frac{\sigma}{\sqrt{n}}$	$sd(B_1) = \frac{\sigma}{\sqrt{\sum (x_j - \bar{x})^2}}$
… Standard Error	$se(M) = \frac{S_Y}{\sqrt{n}}$	$se(B_1) = \frac{RMSE}{\sqrt{\sum (x_j - \bar{x})^2}}$

Sample Mean v. OLS: *Inference I*

2. Inference	**Sample Mean**	**OLS**
Dist. of Y_i	$Y_i \sim N(\mu, \sigma^2)$	$Y_i \mid x_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$
Degrees of Freedom	n-1	n-2
Distribution of Estimator	$\frac{M-\mu}{sd(M)} \sim N(0,1)$ where $sd(M) = \sigma / \sqrt{n}$	$\frac{B_1-\beta_1}{sd(B_1)} \sim N(0,1)$ where $sd(B_1) = \sigma / \sqrt{\sum (x_i - \bar{x})^2}$
t statistic	$\frac{M-\mu}{se(M)} \sim t_{n-1}$ where $se(M) = \frac{S_Y}{\sqrt{n}} = \frac{1}{\sqrt{n}}\sqrt{\frac{\sum (Y_i - \bar{Y})^2}{n-1}}$	$\frac{B_1-\beta_1}{se(B_1)} \sim t_{n-2}$ where $se(B_1) = \frac{RMSE}{\sqrt{\sum (x_i - \bar{x})^2}}$

Sample Mean v. OLS: *Inference II*

2. Inference, continued	**Sample Mean**	**OLS**
Confidence Intervals	$[M \pm c\ se(M)]$, $P[\vert t_{n-1} \vert < c) = p$	$[B_1 \pm c\ se(B_1)]$, $P[\vert t_{n-2} \vert < c) = p$
Null Hypothesis:	$H_0 : \mu = 0$	$H_0 : \beta_1 = 0$
t statistic (under Ho)	$\frac{M}{se(M)}$	$\frac{B_1}{se(B_1)}$
t statistic (for the given sample)	$t_{\hat{\mu}} = \frac{\hat{\mu}}{se(\hat{\mu})}$	$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}$
p value	$P[\vert t_{n-1} \vert > t_{\hat{\mu}})$	$P[\vert t_{n-2} \vert > t_{\hat{\beta}_1})$
Hypothesis Test:	Reject if $\left\vert \frac{\hat{\mu} - 0}{se(\hat{\mu})} \right\vert > c$ or $\vert t_{\hat{\mu}} \vert > c$ … or if $p < \alpha$	Reject if $\left\vert \frac{\hat{\beta}_1 - 0}{se(\hat{\beta}_1)} \right\vert > c$ or $\vert t_{\hat{\beta}_1} \vert > c$ or if $p < \alpha$